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A New Motional Equation of a Gyro Led by Centrifugal Force

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The author aims to analyze the movement direction of the material point of a gyro in precession and calculate the angular momentum by means of centrifugal force caused by the crossing of the direction of centrifugal force and that of a.gyro.

1. I NTRODUCT ION

Until now the precession of a gyro has been explained by $I\omega\Omega=WL$ (I , ω, Ω, W ; I stands for inertia momentum, ωfor the speed of the spinning angle. Ωfor the speed of precession angle, W for the weight of the gyro. L for the length of the rotation axis) , but it does not make sense in that the angular momentum of I ωΩ wi11 increase indefintely, if W and L are made to increase or Ω is made to increase by external force under the condition of constant I ω.

This paper attempts to analyze the movement direction of the material point of a gyro in precession and derive the following equation by means of the couple caused by the inclination of centrifugal force and the momentum caused by the couple.

$$I\,\omega^2\cdot\sin\theta=WL\sin\alpha$$

$$\sin\theta=\Omega\sin\alpha/\sqrt{(\Omega\sin\alpha)^2+\omega^2}$$

α = inclination angle of a rotaion axis to perpendicularity

α =inclination angle of a rotation axis to perpendicularity

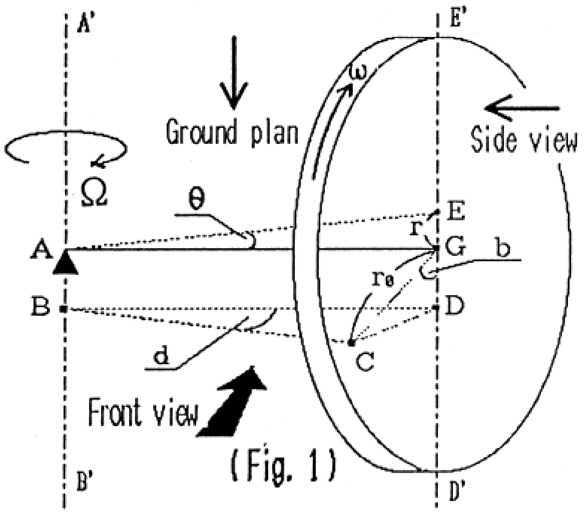
2. DISCUSSION

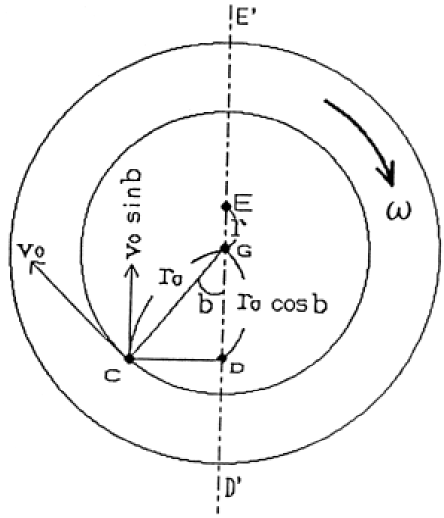
2.1 THE MOVEMENT DIRECTION OF THE MATERIAL POINT OF A GYRO IN PRECESSION

When a gyro is in precession, each material point works in the merging direction of the tangent of the spinning direction of the gyro and that of the movement direction of the gyro in precession.. Its tangent is at a right angle to the momentary spinning axis.

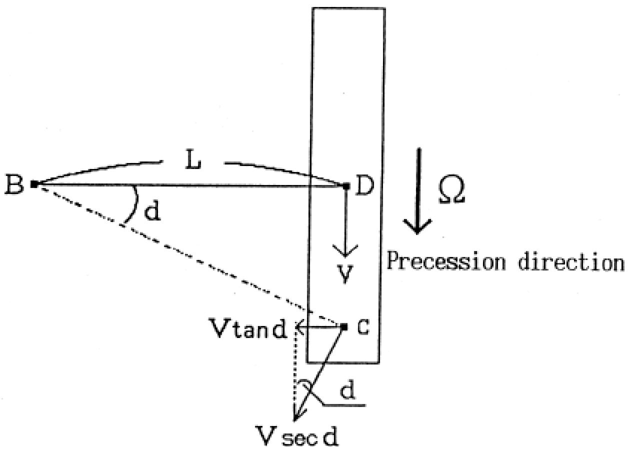
From the following conditions in Fig. 1 :

E : momentary spinning center. G : center of gravity of the gyro. A : fulcrum of spinning axis, line AG : spinning axis, dotted lineAE:momentary spinning axis, angle BDC=right,angle A' AG=right, AB=Gd, AG=Bd, lineA' B' parallel to E' D' . line E' D' :central line which goes through the center of gravity of the gyro and is a right angle to the spinning axis, V : precession speed,
ω : angle speed of a gyro. Va : spinning speed of the material point of radius r_θ,
θ : deviation angle (tentative name--the crossing angle EAG of the spinning axis and the momentary central axis) , r : distance from E (momentary spinning point) to G (central spinning point)

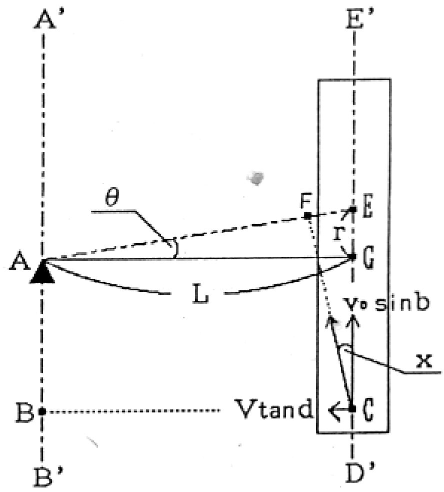




Rotation speed and direction
(Side view of Fig. 1)



Precession speed and direction
(Graound plan of Fig. 1)



(Front view of Fig. 1)

regarding material point C, the following equation is derived :

$V_\theta \sin b$: spinning speed parallel to E' D' in side view
$V \sec d$: precession speed in ground plan
$V \sec d \sin d = V \tan d$: recession speed parallel to spinning axis

The front view is a composite picture of $v_\theta \sin b$ and $V \tan d$.
From the condition of x=angle GCF. The following equation is derived :

$\tan x = r\,\omega \tan d / r_\theta\,\omega \sin b = r \tan d / r_\theta \sin b$ (from $\tan x = V \tan d / v_\theta \sin b$, $V = r\,\omega$, $v_\theta = r_\theta\,\omega$)

$\tan x = L \tan\theta \tan d / r_\theta \sin b$ $\quad L \tan d = r_\theta \sin b$ $\quad$ (from $r = L \tan\theta$)

Therefore $\tan x = \tan\theta$ (Refer to Fig. 1). The crossing angle of the tangent of the material point and momentary spinning axis(AE) is right.

2.2 DEFINITION OF INERTIA RADIUS AND INERTIA QUANTITY

When a homogeneous disc of radius R is spininng on a fixed axis, the following definitions are established:

Definition (1): $r=\sqrt{R^2/2}$ (from $\pi R^2-2\pi r^2=0$ r = inertia radius)

Definition (2): $M=\sum dm$ (from dm = quantity of the material point of the homogeneous disc; M = inertia quantity)

From definitions (1) and (2), the sum total of centrifugal force=$Mr\omega^2$ (ω = angle speed)

2.3 GENERATION OF DEVIATION MOMENTUM(tentative name)

Centrifugal force moves in the direction of the moving material point from the spinning center. From the following condition:

θ =deviation angle (crossing angle) of momentary spinning central axis in precession and spinning axis

couple ($Mr\omega^2 \cdot \sin\theta$) is caused by centrifugal force ($Mr\omega^2$),since each material point moves at an angle of θ to the spinning direction of the gyro.

Deviation momentum is $Mr\omega^2 \sin\theta\, r = Mr^2\omega^2 \sin\theta = I\omega^2 \sin\theta$ ($I = Mr^2$).

and its direction is opposite to the fulcrum of gravity (minus direction) . since the momentary spinning central axis occurs on the spinning axis.

2.4 COMPARISON TO THE TRADITOONAL EQUATION

From the following condition:

r_θ= Iength from the spinning central point of the gyro to the momentary spinning central point

(ω、L、α、Ω、θ、I) the same as above mentioned. The following equations are derived:

$$r_\theta = L\sin\alpha\,\Omega/\omega \quad (\text{from } r_\theta\omega = L\sin\alpha\,\Omega)$$

$$\sin\theta = \Omega\sin\alpha/\sqrt{(\Omega\sin\alpha)^2+\omega^2}$$

$$\cos\theta = \omega/\sqrt{(\Omega\sin\alpha)^2+\omega^2} \quad (\text{from } \tan\theta = r_\theta/L = \Omega\sin\alpha/\omega)$$

$$\begin{aligned} I\omega^2\sin\theta &= I\omega^2\Omega\sin\alpha/\sqrt{(\Omega\sin\alpha)^2+\omega^2} \\ &= I\omega\Omega\sin\alpha\cdot\omega/\sqrt{(\Omega\sin\alpha)^2+\omega^2} = I\omega\Omega\cos\theta\cdot\sin\alpha \end{aligned}$$

When a gyro of gravity W makes a precession at an angle of α to perpendicularity on an axis of length L , gravity causes momentum of $WL\sin\alpha$ to move in the direction of gravity (plus direction) on the fulcrum. Supposing this momentum and deviation momentum are equal, $I\omega\Omega\cos\theta = WL$ (from dividing both by $\sin\alpha$)

When a gyro is making a precession, each material point is making a circular movement at a right angle to the momentary spinning central axis. Therefore, it is assumed that the angle momentum of $I\omega$ is that of the momentary spinning central axis. Supposing the deviation angle is θ, the angle momentum of the gyro on the spinning axis is $I\omega\cos\theta$. Supposing this is right, the equation derived from the traditional angle momentum is the same as that in this paper, and $I\omega^2\sin\theta = WL\sin\alpha$ is defined as the momentum equation of the gyro.

2. 5 LIMIT OF DEVIATION MOMENTUM

$$\lim_{\Omega \to \infty} I\omega^2 \sin\theta = I\omega^2 \quad (\text{from } \sin\theta = \Omega\sin\alpha/\sqrt{(\Omega\sin\alpha)^2+\omega^2} \quad (\alpha \neq 0))$$

2. 6 PRECESSION ANGLE SPEEDΩ AND CONDITIONS OF PRECESSION

$$I\omega\Omega\cdot\cos\theta = I\omega^2\Omega/\sqrt{(\Omega\sin\alpha)^2+\omega^2} = WL \qquad \text{from } \cos\theta = \omega/\sqrt{(\Omega\sin\alpha)^2+\omega^2} \qquad (\alpha \neq 0)$$

From squaring this: $(I^2\omega^4 - W^2L^2\sin^2\alpha)\Omega^2 - W^2L^2\omega^2 = 0$

$$\Omega = WL/I\omega\cos\theta = \sqrt{4(I^2\omega^4 - W^2L^2\sin^2\alpha)W^2L^2\omega^2} / 2(I^2\omega^4 - W^2L^2\sin^2\alpha)$$

Precession condition is $I\omega^2 > WL\sin\alpha$.

3. CONCLUSION

The precession movement equation of a gyro is $I\omega^2 \sin\theta = WL\sin\alpha$. When the direction of a spinning gyro is changed, the momentary spinning central axis crosses the spinning axis of the gyro. This crossing angle (deviation angle) produces the deviation momentum.

Since centrifugal force works at a right angle to the momentary spinning central axis, the deviation momentum occurs in a phase 90 degrees from the point where the force is applied.

The reason a spinning gyro keeps precession on its fulcrum is supposed to be as follows:
static momentum produced by the gravity of a gyro changes into precession direction momentum due to the phase,
deviation momentum begins to work as a result of precession,
equilibrium is achieved between deviation momentum in a negative direction and momentum in a positive direction which work on the fulcrum owing to gravity.

Since deviation momentum works to lessen the deviation angle, a gyro rigid . condition is caused by deviation momentum. The characteristic movement of a gyro is supposed to be caused by deviation momentum.

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Toyoji Baba : About a new motional equation of a gyro led by centrifugal force, Proceedings of The 29th Japan Society for Aeronautical and Space Sciences Annual Meeting, (1998) , PP.. 182-183.